SYMMETRY AND SUBSYMMETRY AS CHARACTERISTICS OF FORM-MAKING: THE SCHINDLER SHELTER PROJECT OF 1933-1942

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This research introduces a formal methodology with an emphasis on the point group symmetry and subsymmetry in the analysis and synthesis of architectural designs. Mathematical techniques, including spatial transformations, a lattice of subsymmetries, and a multiplication table, are reviewed as a formative principle of spatial compositions. The principle is applied to analyze R. M. Schindler's unexecuted work, the Schindler Shelter of 1933-1942, which demonstrates systematical experimentation with symmetric transformations to generate design variations. The principle is also employed to test the compositional possibilities of arranging a shelter on a city block to maximize streetscape variety.
INTRODUCTION

The principle of symmetry prevails in architecture as a design strategy, or compositional methodology, for form-making. It applies to a form as a whole, but it may also apply to components (not necessarily discrete), which make up the form (Weyl, 1952; Rosen, 1975; March and Steadman, 1971; March, 1979; Hargittai, 1986, 1989; Grünbaum and Shephard, 1987; Emmer, 1993). Symmetry also applies to the consequences of multiplying the form in some larger assembly, such as the “room, building, city” analogy used by Alberti in the 15th century with regard to architectural design (Alberti, 1988). Although the final designs may seem asymmetrical as a whole, several layers of symmetry could be manifested in parts of the design, though not immediately recognizable even with a keen eye for symmetry (Park, 2001).

The architect R. M. Schindler (1887-1953) provided an example of form-making that consciously exploited symmetry to generate both interior room variations and multiple house orientations within city blocks. A prime example of this symmetry is the Schindler Shelter project. It was used to illustrate the potential for the conscious application of symmetry in form-making. In particular, it showed how variety might be produced as an outcome of using a unified formative principle. In the same volume, Frank Lloyd Wright and Schindler had independently presented schemes for city residential land development for Chicago in 1914 (Yeomans, 1915). Wright introduced his “quadruple block plan,” or group of four houses. Wright described the proposal as it impacts each householder:

“His building is an unconscious but necessary grouping with three of his neighbors’, looking out upon harmonious groups of other neighbors, no two of which would present to him the same elevation even were they all cast in the same mould. A succession of buildings of any given length by this arrangement presents the aspect of a well-grouped buildings in a park, of greater picturesque variety than is possible where facade follows facade.

(Yeomans, 1915:96-102)

Two years later, as Wright left for Tokyo to work on the Imperial Hotel commission, he wrote a note introducing “Mr. Schindler ... who will have charge of my affairs during my absence” (Sheine, 2001:29).

This study first presents a formal discussion of the mathematical structure of symmetry groups. Then, an analysis is done of symmetry groups in the organization of individual shelter plans as characteristics of form-making. Finally, compositional possibilities of arranging shelters on a city block are demonstrated.

FORMAL DISCUSSION

Because of the potential obscurity of symmetry nested in asymmetrical design, a formal methodology clearly accounting for different hierarchical levels of symmetric employment in architectural designs is needed. In an effort to provide this methodology, a technique founded on the algebraic structure of symmetry groups of a regular polygon in mathematics is presented in this paper (Grossman and Magnus, 1964; March and Steadman, 1971; Budden, 1972; Lockwood and Macmillan, 1978; Martin, 1982; Coxeter, 1986). The methodology will provide not only the description of symmetrical structures of a design but also further insight on generating new designs by combining various symmetries.

Spatial Transformations

Symmetries in this paper are defined as spatial transformations that leave an object in a congruent figure without changing its appearance as a whole. Spatial transformations are called isometries when the transformations preserve the distances between points and angles. There are four kinds of planar transformations: translations, rotations, reflections, and glide reflections (the composite movement of reflections and translations). The set of all isometries of a figure forms a mathematical
structure known as the symmetry group of the figure. It is through the study of these symmetry groups that different types of architectural designs are clearly distinguished with regard to symmetry.

In two dimensions, there are two symmetry groups of plane symmetry: the finite group and the infinite group. The finite group of plane symmetry is called the point group. Spatial transformations take place in a fixed point or line. The transformations involve rotation about the point and reflection along the lines, or the combination of both. In the point symmetry group, no translation takes place. In the infinite symmetry group, spatial transformations occur where the basic movement is either a translation or a glide reflection. In this group, designs that are invariant under one directional translation are called the frieze group, and designs under two directional translations are called the wallpaper group. This paper deals with the symmetries of the point groups in two dimensions only.

In two dimensions, there are two finite point groups: the dihedral group denoted by $D_n$ for some integer $n$; and the cyclic group denoted by $C_n$. The spatial transformations of the dihedral group comprise rotation and mirror reflection. The cyclic group contains transformations by rotation only. The point groups have no translation. The number of elements in a finite group is called its order. The symmetry group of $D_n$ has order $2n$ elements, while $C_n$ has order $n$ elements. For instance, the symmetry of the square, which is the dihedral group $D_4$ of order 8, has eight distinguishable spatial transformations that define it: four quarter-turns and four reflections — one each about the horizontal and vertical axes and the leading and trailing diagonal axes. $C_4$ has four spatial transformations: the four quarter-turns.

Valid operations of a symmetry group of the square include rotation about its center through 90, 180, 270, or 360 degrees, and reflection on its four axes. Eight distinguishable transformations of the symmetry group are labeled as $I$, $q$, $q^2$, $q^3$, $r$, $qr$, $q^2r$, and $q^3r$. In our notations, "$I$" denotes identity; "$q$" denotes a quarter-turn clockwise rotation of the square; "$q^2$" denotes a half-turn clockwise rotation of the square; and "$r$" denotes a mirror reflection of the figure. The diagrams in Figure 1 illustrate the exact positions of the figure. Since there cannot be any more than 8 symmetries, each symmetry should determine exactly one position for each corner.

FIGURE 1. Eight distinguishable transformations of the square where "$q$" represents a quarter-turn clockwise rotation and "$r$" represents a mirror reflection.

FIGURE 2. Lattice showing the order of subsymmetries from the complete symmetry of the square at the top and the asymmetrical identity at the bottom.
**Lattice of Subsymmetries of the Square**

Subsymmetries arise from a curtailment of some symmetry operations: that is, formally selecting subgroups from the group of symmetries. We examine the lattice of subsymmetries of the square ($D_4$). Figure 2 illustrates all possible subsymmetries in a hierarchical order; some with four elements; some with two; and only one, the identity or asymmetry, with one element.

Starting from the top of the diagram, Level 1, the full symmetry of the square, $D_4$, forms the superimposition of the eight distinguishable spatial transformations that comprise this group, including four quarter-turns and four reflections, one each about the horizontal and vertical axes and the leading and trailing diagonal axes.

Level 2 consists of two reflexive subsymmetries ($D_3$). One shows two orthogonal axes ($D_{2h}$), and the other shows two diagonal axes at $45^\circ$ to the orthogonal ($D_{2d}$). Both of these subsymmetries exhibit a half-turn through $180^\circ$. The $D_{2h}$ subsymmetry exhibits vertical and horizontal reflections and a half-turn. The $D_{2d}$ subsymmetry exhibits reflections about the leading and trailing diagonals, as well as a half-turn. The third subsymmetry shows four quarter-turns, $C_4$, or $90^\circ$ rotations. The design of $C_4$ forms the superimposition of four spatial transformations: the four quarter-turns with no reflection.

At Level 3, there are five subsymmetries. Four of these subsymmetries include reflective symmetry $D_1$; two subsymmetries with a single reflective axis on the orthogonal simple bilateral symmetry ($D_{1v}$ and $D_{1h}$) and two subsymmetries with a single reflective axis on the diagonal ($D_{1d}$ and $D_{1t}$). The $D_{1v}$ subsymmetry exhibits a reflection about the vertical axis only, while the $D_{1h}$ subsymmetry exhibits a reflection about the horizontal only. The $D_{1d}$ subsymmetry exhibits a reflection about the leading diagonal only, while the $D_{1t}$ subsymmetry exhibits a reflection about the trailing diagonal only. The fifth subsymmetry at this level, $C_2$, includes only the half-turn rotation. This element has no reflection axes and no rotation less than the full-turn through $360^\circ$. At the bottom level is the unit element, or the identity of the group, $C_1$, which shows asymmetry, that is, no reflections and no rotations.

As with the examples of regular polygons, such as an equilateral triangle, a pentagon, and others, the subgroups may be further differentiated according to axes into what we are calling here their subsymmetries (Park, 2000). A polygon with $n$ edges has, at most, dihedral symmetry of order $2n$, where the order of a finite group is the number of elements. The subgroups of the symmetry group of a regular $n$-gon are perhaps ordered in the lattice diagram. For instance, $D_3$ is the group of symmetries of an equilateral triangle, which has order 6 with its $D_1$, $C_3$, and $C_1$ subsymmetries.
Multiplication Table

By multiplying two symmetries of the regular polygon, it is possible to derive another symmetry of the figure. For example, let us combine "q" and "q'r" of the symmetry of the square. The notation [q][q'r] means mirror reflection [q'r], then rotation [q], reading from right to left. Since [q][q'r] is a symmetry of the square, the effect must be one of the 8 symmetric transformations. In fact, the effect of [q][q'r] is the same as [q'r]. If those two elements are combined in a reverse order, then [q'r][q] not the same as [q'r}], but [q'r]. Here, the multiplication order is important. The multiple computation of the symmetry of the square yields a singular result. For instance, "qq'rqq'r" is a particular symmetry of the square, so it must be one of those eight. In fact, it is the same as "qr." Thus, any computation of the symmetries of the square concludes with one of those eight different transformations.

At this point, it is appropriate to define a complete structure of sym-
In this section, the discussion focuses on the extent to which the formal methodology described above may apply in the analysis of an architectural design and the synthesis of multiple arrangements of the design on a city block. The methodology will be applied to the Schindler Shelter developed between 1933 and 1942 by R. M. Schindler. The design was a reaction to the low-cost housing projects for the Subsistence Homesteads intended to provide urban dwellers with an opportunity to obtain economic security, as well as comfortable suburban shelter (Park, 1999). Schindler responded to the program with issues of flexibility of the floor plan, expandability for the changing needs of a growing family, minimum maintenance, new construction methods, and new materials. A key factor in his proposal
was to provide a variety of optimum space layouts with the integration of composition and construction. Schindler developed various designs based on two different types of construction systems: the Shell Construction System and the Panel Post Construction Method. Since schemes of the Panel Post Construction Method derive their basic composition from those of the Shell Construction, this analysis is confined to the schemes of the Shell Construction System. The archival research verifies that while a series of shelter plans undergo a variety of spatial transformations for many years, they underlie a unified formative principle.

The Spatial Organization of the Project

This discussion focuses primarily on extracting underlying principles of spatial organization from the designs. Then, a series of schemata is constructed to describe its compositional order. Although these schemata may be radically reductive and excessively simple, they clearly define the compendium of design logic.

The overall parti of the shelter plans with the Shell Construction System is set out over a 5 x 5-foot unit square, although a 4 x 4-foot unit square was used in laying out the plans of the Panel Post Construction Method. The architect clearly marked the unit on the drawing (Figure 7a). Along with the unit grid, a pinwheel type of symmetry governed the internal structure of functional zones in each scheme (Figure 7b). Overlaying the unit grid, the transparent interplay of a variety of symmetric principles was unique and eminent in Schindler’s designs (Park, 1996, 2000, 2001).

The kitchen, bathroom, and laundry were allocated as a unit group to concentrate the plumbing systems into a single wall (Figure 8a). Using this grouping, supply lines, waste branches, and soil pipes were simple and short so that the plumbing stack would be saved. Laundry was provided in an open porch, which afforded an excellent means of open-air drying. The rest of the house was a one-piece shell. The shell was subdivided into three major rooms. One room was the living room, which extended a unit module (5-foot) outward from the basic parti of the house for spatial necessity (Figure 8b). The extended module contained the fireplace and the main entry of the shelter. The other two rooms included removable closet partitions for flexible expansion according to the changing needs of a growing family. The removable closet partitions were set along the pinwheel type of symmetry, making the central space a hall. The closet was spacious enough and opened alternately into one room or the other. The clerestory windows above the central hall helped to light and ventilate rooms. The bathroom was adjacent to the central hall, allowing it to be well-ventilated.

The garage was a separate unit that could be added to any side of the house and that provided spatial variety from the exterior view (Figure 9). The garage was large enough to serve as a workshop and
storage. Its rooftop provided space for sunbathing. The additional living space and the garage broke the solid symmetrical juxtaposition of the core parts but produced asymmetrical designs. Breaking the symmetry exerted another mechanism for the creation of a variety of designs, reinforcing the dynamic exterior view.
Based on archival resources, there exist four different types of shelter plans, as well as their variations. Their differences are based on the number of room types, for example, 3-, 4-, 4⅓-, and 5-room type, as defined by Schindler.

**Design Variations and Their Arrangement**

Since the development of a prefabricated housing scheme and its mass production may result in a monotonous appearance, variations of the prototypical design are important. The Schindler Shelter designs achieve the quality of the variations, retaining their unified principle and remaining unchanged as symmetry transformations take place. The designs also increase a variety of the exterior appearances with the garage location. That is the novelty of the Schindler Shelter. Six variations of a 4-room type scheme were identified by the investigation of the archival drawings. The basic room type was rotated and reflected. Then, a garage was added to a side of the unit. The variations looked different, but a closer observation demonstrated that they were almost identical, based on a unified formative principle.

The generation of variations presents further questions on how to arrange the variations in the planning of larger projects. Although the variations could be assembled in a variety of ways, including end-on units, courts, clustering patterns, or pinwheel format of court patterns, the architect of the Schindler Shelter provided only the street-front pattern of six housing arrangements as an example. The shelters lined both sides of the street, providing independent and easy access to the shelter. His example reflected and rotated only a 4-room type shelter. Garages were added in different locations. No garage was attached to the shelter in the same location as another shelter. Shrubs bordered each lot property, providing its own front and back yard. Schindler demonstrated that the change in location of the garage and variations of the landscape layout created different street-front designs.

Obviously, when a standard unit and its variations are mixed in different combinations, the possibilities of their groupings will be considerably increased. Schindler demonstrates that minor variations of the architectural theme in each unit provide differing identities for each dwelling, diminishing its monotonous character.

Earlier experiments of symmetric juxtaposition of a housing unit in a large assembly are found in various housing examples — most notably, the Monolith Homes for Thomas R. Hardy (1919), the Pueblo Ribera (1923), and the Harriman project (1924-1925). The Monolith Home stands out as one of the earliest housing experiments of this kind. It was developed for F. L. Wright in Racine, Wisconsin, while Wright was in Tokyo working on the Imperial Hotel. The spatial composition of the unit plan was set along the strong cross axis. Then, the typical unit plan was assembled in four quarter-turns, or 90° rotations, in a larger assembly. The same idea was applied to the Pueblo Ribera Court. There was no strict symmetry involved in the standard unit design, yet the units were grouped.
and arranged in four quarter-turns "to obtain a combination (that) achieves architectural form as a whole" (Schindler, 1949). Subsequently, the idea continued in the unrealized housing project for J. Harriman (Gebhard, 1980).

The Combinatorial Possibilities of Arranging the Schindler Shelters on a City Block

A central issue with regard to the analysis of the project is the synthetic aspect of design. While a variety of shelter designs with the unified compositional principle may exist, the architect never thoroughly explores all possible layouts. He only provides some examples of the kind of layouts that he developed. An exhaustive number of plans could be generated as a complete map of probable designs. In generating combinations of possible layouts, symmetry principles as characteristics of form-making are continuously employed in the design upon which variations are created.

The application of enumeration principles results in huge compositional possibilities. For example, by manipulating both the 3- and 4-room types, new designs are generated by simply rotating and reflecting the standard shelter plan and adding the garage, while still maintaining formal continuity among them. The designs below are completely new, but adhering to Schindler’s principles could
form a possible family of the Schindler Shelter.

This idea could be extended to regroup the shelters in a speculative compartmentalization of a city site. To illustrate, the shelters on a city block are arranged in a way Wright envisions in his Quadruple Building Block (Wright, 1994). In this example, a hypothetical square lot is represented by the basic block. On the lot, a simple dot represents the site of the garage location and driveway for the orientation of the garage. The dot corresponds to the symmetry group of the square. Each block holds eight households clustering around a semi-private open space, like Gill’s Sierra Madre project of 1910 (McCoy, 1960). Then, eight blocks are regrouped in a larger assembly that includes an open community space located in the middle. Not only can a shelter be arranged corresponding to one of the eight elements in the symmetry group of the square, but variations of different types of the shelters can also be distinctively combined to generate a huge variety of arrangements, resulting in a variety of streetscapes. All locations of the shelters with their proper positions are shown in Figure 14.

Figure 15 is a detailed example of how variations of shelters can be arranged. In this example, 3- and 4-room types of the shelters are arranged in one of the eight clusters. Eight shelters are grouped within the square lot sharing a central semi-private yard. The open yard provides a well-defined play area for children, as well as a recreation area for the resident. Each garage attached to, or detached from, the individual dwelling is connected to the road by a driveway.

In the layout, pergolas, a cantilevered entrance or deck, and a built-in flower box could be attached to each unit, as Schindler suggested. A row of trees around the entire block at the sidewalk and trees or shrubs between the units could also be added to provide visual protection, maintain privacy, provide a relaxing view, and define the border of exclusive space for each lot. The rich landscape could include shrubs as high as six feet to provide desirable sunlight and privacy.

When illustrated in a three-dimensional perspective, the layout would provide a dynamic look in the streetscapes as demonstrated in the bird’s eye view of the eight-unit cluster (Figure 16). Eye-level street scenes of the block from each of four corners (Figure 17) provide a glimpse of what the development might look like to someone walking by at eye-level (5-foot 6-inch) or driving by in a car (4-foot). Despite the substantial geometrical similarities of each unit, the streetscape of each side of the block is distinctive, possessing only a few common configurational properties from the pedestrian’s perspectives. Walking along the streets, pedestrians should enjoy the variety of the street scenes and a minimization of monotony. The explanation and examples presented in this paper demonstrate how symmetry, with its variations, can be used to create diversity in streetscapes.

CONCLUSION

This paper has explained symmetry and the subsymmetry methodology of the point group as characteristics of form-making. It serves as a guide to illustrate the specific applications of symmetry groupings in the analysis and synthesis of architectural designs. The Schindler Shelter designs are analyzed
FIGURE 16. A perspective rendering showing a variety of three-dimensional looks in the streetscapes.

FIGURE 17. Four eye-level perspective views from each corner of the site.
with regard to the apparent uses of various symmetries in individual designs and their variations, albeit ambiguous or invisible when seen as a group. The dynamic combinations of symmetric transformations and the external garage lead to unique integral organizations of spatial form.

The garage annexation remarkably augments the performance of symmetric transformations in the creation of design variety. Compositional ideas defined in the analysis are used in the generation of new designs where a myriad of possible designs could be produced while preserving a unified formative principle. In this respect, Schindler’s approach excels.

This paper also demonstrates how shelter designs based on symmetric transformations can be clustered on a hypothetical city block. Giving credence to F. L. Wright’s quadruple building block, this study demonstrates that the symmetric transformations of a shelter and its arrangement on a city block could produce a maximum variety of external facets from any side of the street.

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