Subsymmetry analysis of architectural designs: some examples

Jin-Ho Park
Department of Architecture and Urban Design, University of California, Los Angeles, CA 90095, USA; e-mail: jinhpark@hawaii.edu
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Abstract. An analytic method founded on the mathematical structure of symmetry groups is defined and some applications to the analysis of architectural designs are shown.

In earlier work by March and Park, architectural designs were analyzed with respect to a partial ordering of subsymmetries associated with the symmetry of the square and then classified by lattice diagrams of the subsymmetries. The analytic approach demonstrates how different subsymmetries may be revealed in each part of the design and how various symmetric transformations combine to achieve the whole design. At first glance, the individual designs seem intricate and without obvious symmetry. However, an analysis of the subsymmetries and symmetric transformations clearly exposes the underlying structure. In this paper, the methodology employed in previous papers is substantially recounted, but new architectural examples have been added.

The methodology of subsymmetry analysis
In the methodology, various types of symmetry, or subsymmetries, are superimposed in individual designs and illustrate how symmetry may be employed strategically in the design process. An account of the mathematical structure of symmetry groups in analyzing architectural designs has been given over the last ten years by Lionel March in his graduate lectures at UCLA on the Fundamentals of Architectonics: Symmetry (reading materials for this course include Baglivo and Graver, 1976; March, 1995; March and Steadman, 1971; see also Budden, 1972; Grossman and Magnus, 1964; Grünbaum and Shephard, 1987; Shubnikov and Koptik, 1974; Weyl, 1952). His emphasis has two aspects. Analytically, by viewing architectural designs in this way, symmetry which is superimposed in several layers in a design and which may not be recognizable immediately may be articulated and made transparent. Synthetically, he believes that architectural design—at its best—and mathematical knowledge are intimately connected and that the design can benefit only by being conscious of group operations and spatial transformations associated with symmetry in compositional and thematic development. March (1995, page 16) cites F W J Schelling's remark given in his lectures at Jena in 1801 and 1804 (Stott, 1989, page 165): “Architecture necessarily proceeds in its constructions according to arithmetical or, since it is music in space, geometric relationships.”

To begin with, it is necessary to provide an elementary account of the mathematical structure of a symmetry group, in particular the point groups in two dimensions. In two dimensions, there are two finite point groups: the dihedral group denoted by D_n for some integer n, and the cyclic group denoted by C_n. The spatial transformations of the dihedral group comprise rotation and mirror reflection; yet the cyclic group contains rotation only. The point groups have no translation. The number of elements in a finite group is called its order. The symmetry group of D_n has order 2n elements, whereas C_n has order n elements. For example, the symmetry of the square which is the dihedral
group $D_8$ of order 8 has eight distinguishable spatial transformations which define it: four quarter-turns; and four reflections, one each about the horizontal and vertical axes and the leading and trailing diagonal axes. The cyclic group $C_4$ has four spatial transformations: the four quarter-turns.

Now let us consider the lattice of subsymmetries of the square (figure 1).

The symmetry group of a square $D_4$ includes not only reflections in its four axes but also rotations through 0°, 90°, 180°, and 270°, respectively. Thus the symmetry group of the square contains eight transformations, and these are the elements of the group. The diagram illustrates all possible subsymmetries: some with four elements, some with two, and just one, the identity of asymmetry, with one element. The structure of the diagram can be accounted for in two ways: from top to bottom, symmetries are 'subtracted' from the full symmetry of the square; and conversely, from the bottom to the top, subsymmetries are 'added' to achieve higher orders of symmetry. Such a reading is analogous to a lattice diagram of subsets of a set, or subshapes of a shape.

Figure 1. The lattice of subsymmetries of the square. At the top is symmetry $D_8$ of order 8, below are subsymmetries of order 4, then below that again are subsymmetries of order 2, and finally subsymmetry $C_1$ of order one, the unit element.

Starting from the top of the diagram, level 1 represents the full symmetry of the square $D_8$ with four rotations and four reflections. Level 2 consists of two reflexive subsymmetries $D_2$: one shows two orthogonal axes, and the other shows two diagonal axes at 45° to the orthogonal. Both these subsymmetries exhibit a half-turn through 180°. The third subsymmetry shows four quarter-turns $C_4$, or 90° rotations: it typifies the 'pin-wheel' symmetry used by architects. At level 3, there are five subsymmetries. Four with reflective symmetry $D_4$, two subsymmetries with a single reflective axis on the orthogonal, simple bilateral symmetry, and two subsymmetries with a single reflective axis on the diagonal. The fifth subsymmetry $C_2$ at this level has the half-turn rotation only. At the bottom level is the unit element or the identity of the group $C_1$. This element has no reflection axes, and no rotation smaller than the full-turn through 360°.

In general, the basic features of the subsymmetries of the regular polygons can be classified. As with the example of the square discussed above, the subgroups may be further differentiated according to axes into what I will call here its subsymmetries [Economou (1997) gives a further mathematical discussion of the technical issues
Table 1. The relation between a regular n-gon, its symmetry group and order, and the lattice diagram of its subgroups.

<table>
<thead>
<tr>
<th>$D_n$</th>
<th>Order $(D_n)$</th>
<th>Polygon</th>
<th>Lattice diagram</th>
<th>$D_n$</th>
<th>Order $(D_n)$</th>
<th>Polygon</th>
<th>Lattice diagram</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_1$</td>
<td>4</td>
<td>$D_2$</td>
<td>$D_1$</td>
<td>$D_6$</td>
<td>16</td>
<td>$D_3$</td>
<td>$D_1$</td>
</tr>
<tr>
<td>$D_2$</td>
<td>6</td>
<td>$D_3$</td>
<td>$C_1$</td>
<td>$D_6$</td>
<td>18</td>
<td>$D_3$</td>
<td>$D_1$</td>
</tr>
<tr>
<td>$D_3$</td>
<td>8</td>
<td>$D_4$</td>
<td>$C_1$</td>
<td>$D_{10}$</td>
<td>20</td>
<td>$D_3$</td>
<td>$C_1$</td>
</tr>
<tr>
<td>$D_4$</td>
<td>10</td>
<td>$D_6$</td>
<td>$C_1$</td>
<td>$D_{10}$</td>
<td>24</td>
<td>$D_3$</td>
<td>$D_1$</td>
</tr>
<tr>
<td>$D_5$</td>
<td>12</td>
<td>$D_8$</td>
<td>$C_1$</td>
<td>$D_{12}$</td>
<td>30</td>
<td>$D_3$</td>
<td>$D_1$</td>
</tr>
<tr>
<td>$D_6$</td>
<td>14</td>
<td>$D_10$</td>
<td>$C_1$</td>
<td>$D_{16}$</td>
<td>32</td>
<td>$D_3$</td>
<td>$D_1$</td>
</tr>
</tbody>
</table>

involved in making these distinctions]. A polygon with $n$ edges has at most dihedral symmetry of order $2n$. The subgroups of the symmetry group of a regular $n$-gon are ordered in the lattice diagram (table 1).

**Some examples of subsymmetry analysis**

**Pantheon at Rome**

The Pantheon was built by Hadrian and was completed by 121 AD. At a glance, the ground plan of the principal space is seen to be a simple centralized design. On closer examination, it can be seen that the underlying composition of the rotunda is based on a 16-gon within the perimeter circle (figure 2, see over). This foundational geometry guides the major decisions of the formal and spatial composition as well as the architectural details. For further architectural studies of the Pantheon, see Licht (1968) and Stierlin (1984).
Figure 2. Pantheon, Rome. Subsymmetry analysis (March, 1998). At the top, the symmetry $D_{32}$ of order 32 is manifest in the 16 divisions of the circle which determine the development of the detailed plan. $D_{32}$ contains four dihedral subgroups: $D_8$, $D_6$, $D_4$, $D_2$ (table 1). The Roman architects of the Pantheon did not choose to exploit the cyclic symmetries. With the eight inscribed angular niches and the eight double columns in front of the small rectangular niches, the diagram at the next level illustrates $D_8$ symmetry. This represents the full symmetry of the octagon. The diagram at the level below shows the $D_4$ symmetry of the square. It includes four large rectangular exedrae, double columns, circular and rectangular floor pavings. The diagram at the next level below indicates $D_4$ symmetry consisting of two orthogonal reflective axes. It includes two additional semicircular exedrae. The diagram at the bottom illustrates $D_2$ symmetry containing the front pronaoos, the entrance into the rotunda, and the back chambers with the largest exedra diametrically opposed to the entrance.
Thus a sequential analysis of the ground plan of the Pantheon reveals how various subsymmetries of symmetry $D_8$ are systematically superimposed. The use of $D_8$ in modern application is also noticeable in Louis Kahn's National Assembly building in Bangladesh (Futagawa, 1994; Kahn, 1988). In this project, Kahn exploits the subsymmetries of $D_8$ in the development of the spatial composition of the design. For example, the inner perimeter of the assembly chamber begins as a 16-gon which is expressed in the ceiling plan. This high-order symmetry is reduced to $D_4$ with the introduction of stairs and seating areas. Four office blocks are placed diagonally around the assembly hall to produce $D_4$ symmetry. The Entrance Hall, the Recreation Room, the Chamber of Ministers, and the Hall of Debates are each characterized by $D_4$ symmetry. The Mosque oriented towards Mecca is marked by its break from the axial symmetries of the secular structure, and its placement ensures that the whole design is actually asymmetrical despite an almost obsessive concern for symmetry in the parts, including the Mosque. Further analyses show that all the dihedral, but none of the cyclic, subsymmetries of $D_8$, may be identified in Kahn's design.

Unlike its floor plan, the cupola of the Pantheon is comprised of 28 radial divisions, with coffers in five horizontal bands. Licht (1968) claims that the number 28, being the sum of its factors $1 + 2 + 4 + 7 + 14 = 28$, is derived from Euclid's definition of a perfect number (Heath, 1923: Book VII, Definition 22): however, he finds "no direct relation to the other proportioning of the building". March (1998) points out that the Euclidean theory of perfect numbers was restated by two mathematicians, Nicomachus (D'Ooge, 1938) and Theon of Smyrna (Lawlor and Lawlor, 1979), who wrote in the second century, contemporaneously with the construction of the Pantheon. Nevertheless, he suggests, the source may be Vitruvius, or at least the tradition that he both recorded and initiated. Vitruvius (Granger, 1933, pages 161–163) distinguishes between the Euclidean perfect number 6, $6 = 1 + 2 + 3$, and the Platonic number 10, $10 = 1 + 2 + 3 + 4$, which is not perfect in the Euclidean sense but which was given special status by the Pythagoreans as the *tetraconta*. The number 16 was produced according to Vitruvius (Granger, 1933, page 165) by 'throwing' both 6 and 10 together to make the most perfect number: "sex et decem, utrosque in unum coe携程nt et fevertur perfectissimum decusis sexis". The number 28 counts the nights of the lunar cycle described in some detail by Vitruvius (Granger, 1935, page 217) without noting its arithmetical perfection. This suggests that the number 16 may be derived from an earlier discussion by Vitruvius (Granger, 1933, page 65) on the disposition of the Roman winds. He counts 8 principal winds but talks somewhat ambiguously about dividing the circle into 16 parts. The ground plan of the Pantheon marks out the 'compass' points where the earthly winds blow with the entrance facing precisely due north. Above, in the cupola, the heavens rule over a period of 28 nights with the lunar cycle, described by Vitruvius in terms of 'quarters', the common factor between 16 and 28. March (1998) points out that the proportional relation 28:16:7:4 between the plan and the cupola divisions is a rational convergent to the side of an equilateral triangle of length 7 inscribed in a circle of radius 4. Such a figure is evoked by Vitruvius (Granger, 1933, page 21) where he notes a question held in common between astronomers and musicians "de sympathia stellarum et symphoniarum in ... trigonis", and again in describing the design of a theater based on four equilateral triangles inscribed in a circle "in duodecim signorum caelestium astrologia ex musica conveniencia astroorum ratioeineratur" (Granger, 1933, page 283). Here is a musico-astrological reference to the 'trine' which Ptolemy was to develop in greater detail in *Harmonics* (Barker, 1989; Book II, pages 380f) written in Alexandria most probably at the time the Pantheon was under construction in Rome. The repeated $4 + 3 + 1 + 4 + 2 + 2 + 4 + 4 + 3 + 4$ polyrhythmic interplay of these two divisions $7 + 7 + 7 + 7$ and $4 + 4 + 4 + 4 + 4 + 4 + 4$ is shown in figure 3.
Ward Willits skylight of Frank Lloyd Wright

Wright's skylight design of 1902 has been interpreted by March (1995) in the context of the kindergarten method, in particular, the flat tiles of Froebel's seventh gift. He presents analytic examples and constructive possibilities of using the gift. In his discussion of Froebel's 'forms of beauty' the possibilities of various symmetric patterns are suggested. Using the gift, especially the scalene triangle which is the only one of five tiles not to possess symmetry, it is possible to draw an ordering of subsymmetrical designs. These designs exhibit symmetry of $D_{12}$ of order 24 and its subsymmetries (figure 4).

Wright's Willits skylight design is examined in terms of its subsymmetries. The design shows Wright's appreciation of symmetry and the spatial relations identified with the forms of beauty (figure 5).

The Free Public Library project of R M Schindler

In this analysis I will examine an ordering of subsymmetries associated with the symmetry of square by scrutinizing the original drawings of Schindler's Free Public Library competition project of 1920. A fuller interpretation of the project and historical influences leading to its design are discussed in Park (1996). Schindler’s ‘Reference Frames in Space’ (March, 1994a; Schindler, 1946) and diagonal symmetry are integrated to act as the governing principle of the scheme. The architect’s original intention to use a diagonal parti is clearly shown in the thumbnail sketch which he scribbled on the back cover of his copy of the competition program (figure 6, page 128).

This sketch places the building at the innermost corner of the lot and anchors the diagonal axis at this point. Two exterior ramp areas, and two parts of the stack room (figure 7, page 128) are arranged symmetrically along this diagonal axis.

The analytic diagram of each floor plan is classified according to the lattice of subsymmetries of the square $D_4$. As a whole, the basement floor-plan shape does not possess the full symmetry of a square but there are some conforming subshapes within it. Other subshapes conform to other subsymmetries. By extracting subshapes which maximize the representation of a particular subsymmetry of the square, we can construct a diagram to illustrate the overlay of symmetries involved in the floor plan at each level. The analysis has been repeated for the four floor levels including the balcony level (Park, 1995) but only the main floor plan is illustrated here. It is noteworthy that the purely rotational subsymmetries, $C_4$ and $C_2$, are not present. Neither the quarter-turn nor the half-turn symmetries are to be found in subshapes without also being accompanied by reflections. Technically this means that in this project
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Figure 5. Skylight from the W Ward Willetts House by Frank Lloyd Wright, Highland Park, 1902. The top part shows the full symmetry of the square $D_4$. The second row shows the symmetry of a rectangle $D_2$ (not illustrated in March, 1995), and the whole design with the pinwheel symmetry $C_i$. In the third row, the diagram shows two designs that are symmetrical along either the vertical axis or the horizontal axis $D_1$, and to the right a reflective symmetry along the diagonal axis, also $D_1$. The bottom row shows an individual panel with asymmetry $C_1$. The lattice of subsymmetries is shown on the right with those used by Wright indicated in black (after drawings by Hyunho Shin).
Schindler uses dihedral, but not cyclic, point symmetry (figure 8, page 130). A similar diagram may be constructed for each floor level, including the basement and the roof. The balcony level is particularly compelling because it is contained within a square envelope with a double-height square void in the center. The second floor reasserts the predominance of the diagonal axis by being contained by a thick L-shaped envelope.

This Library project represents a significant turning point in Schindler's architectural career because its design explores and clarifies his formal methodology. It is apparent that Schindler's emphasis on diagonal symmetry in this public project is uncommon for the period. Further, his layering of subsymmetries is executed with masterly skill. He continued to develop the use of the diagonal symmetry in his later works, most notably the How House of 1925. The list might also include residences for Elizabeth van Patten in 1934–35 and Mildred Southall in 1938, a beach house for Olga Zacek in 1936–38, the residence of A van Dekker in 1940, the Bethlehem Baptist Church in 1944, and the Kallis House in 1946 (see Gebhard, 1997; Sarnitz, 1986). Moreover, there are some modern architectural examples of the use of diagonal symmetry just after Schindler’s Library project. Erich Mendelsohn, Schindler’s exact contemporary, for whom Richard Neutra was working at the time he was corresponding with Schindler (McCoy, 1979), explored the use of diagonal symmetry in the Double House of 1922. Also at this time, Wright—for whom Schindler had been office supervisor—developed various textile block patterns for his Los Angeles residences. In the Ennis House of 1922–24, the pattern of the unit concrete block is characterized by a strong diagonal. Closer examination shows that the pattern design is slightly off axis.

The How House of R M Schindler

The How House stands out from Schindler’s other works of this period in its conspicuous and transparent play around the diagonal axis overlaying a 48-inch unit system (March, 1994b; Parlee, 1987; Schindler, 1929; 1993; Sheine, 1998a; 1998b; Steele, 1998a; 1998b; 1998c; 1998d; 1998e). The relationship between the unit and its subsymmetries is particularly clear in the How House, where the unit contains a hole corresponding to the diagonal axis of subsymmetry.

Figure 6. The Free Public Library by R M Schindler, Jersey City, 1920: sketch design (Computer-enhanced redrawing from original in the Architectural Drawing Collection, University of California, Santa Barbara, CA).

Figure 7. The Free Public Library by R M Schindler, Jersey City, 1920. Axonometric of the book stacks reflected along a diagonal axis.
Figure 8. The Free Public Library by R M Schindler, Jersey City, 1920. In the lattice diagram of the first-floor plan, the subshapes of the plan are partially ordered with the full symmetry of a square of order 8 at the top. At the next level, there are two reflective symmetries of order 4—one is orthogonal along two axes. At the level below, there are four reflexive symmetries of order 2, each along a single axis, either orthogonal or diagonal. At the bottom of the diagram is the floor plan itself, which represents ‘identity’. The diagram illustrates how various symmetries are superimposed.

1996; Taut, 1929, pages 178 – 179). It appears that the How House derives its compositional themes from the unbuilt Library project. The use of both the modular and the symmetrical designs in the How House is subtle (figure 9, see over).

Schindler increases the significance of the diagonal axis by setting the orthogonal lines of the ground plan at a 45° angle to the boundaries of the lot and the road frontage [figure 10(a), page 131]. The lower portion of the building volume is built in concrete with Schindler’s own ‘slab-cast’ construction system: the structure of the upper portion is redwood.\(^1\) The horizontal stratification of the continuous concrete

\(^{1}\) In Modern Architecture Henry-Russell Hitchcock (1929, page 213), agreeing with J J Oud, wrote that such combinations of material were “unsuited to the symbolism of the new manner by their irregularity and natural character, that is in a sense their ‘picturesqueness’. Then in reference to the How House which he had only viewed in print, probably a preview of Schindler’s 1929 article, he continues “This is clearly seen in certain Belgian houses in brick, and in particular in Schindler’s premature attempt to place a wooden superstructure on a concrete base in 1928 [sic], although the result was, all the same the best house he has so far built.”
course as well as the battenboards on the wall of the house were laid to coincide with a 16-inch vertical module. The module incorporates the heights of all elements of not only the main structure but also the built-in furniture, chairs, windows, and doors. In Schindler's words, this provides 'a uniform scale'.

Whereas the Library Project is composed by the superimposition of a variety of subsymmetries, the How House is determined mainly by reflective symmetry along a diagonal axis. In the words of Schindler (1929, page 5), "the rooms form a series of right angle shapes placed above each other and facing alternatingly north and south". The analysis focuses on the piano nobile—the living room, the dining room and Dr How's study—where the spatial and structural setting of the whole composition is based on the diagonal axis. However, the floor plan of the piano nobile does not conform to symmetry along the diagonal axis in a strict manner: the symmetry is broken by additional spaces such as the kitchen and the entrance hall, and also in certain details such as the principal fireplace and the stairway to the lower, bedroom floor. This asymmetry does not depend on the arbitrariness of personal taste which rejects the principles of symmetry in the name of modernism. Instead the asymmetrical design is generated from a disciplined understanding of symmetry. The final design displays an abundance of symmetries within the parts while negating the strict symmetry of the whole.

There are two structural layers at the gallery level, \( \frac{1}{4} \) module apart: the lower layer which has the same herringbone pattern on the ceiling of the porch extending over the terrace as the pattern on the living room ceiling [figure 10(b)]; and the upper layer which splits into two wings over the dining room and the study, respectively [figure 10(c)]. Both L-shaped layers are set along the diagonal axis facing south and north in opposite orientations to each other. The lower part, in particular, demonstrates the ingenuity of

**Figure 9.** The How House by R M Schindler, Los Angeles, 1925. Axonometric of the structures defining the piano nobile.
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Figure 10. The How House by R M Schindler, Los Angeles, 1925. (a) The site plan. (b) Gallery level including the porch over the terrace. (c) Ceiling over the dining room and study with aisle extensions into the corner of the living room. (d) Ceiling structure over the living room.

Figure 11. (a) Ceiling of the living room in the How House; (b) pin-wheel structure spanning distances greater than the beam length; (c) structural scheme by the renaissance architect Sebastino Serlio; (d) structural theme by the medieval master mason Villard de Honnecourt.
Schindler’s method which causes two partially cantilevered beams to cross above the open shaft area without any support at the center.

The exposed ceiling structure of the living room fills a square plan with a span of 20 feet, or five 48-inch modules. Two chimney stacks on adjacent sides emphasize the diagonal axis [figure 10(d)]. The structure of this ceiling is a system of redwood beams at \( \frac{1}{2} \)-module intervals (24 inches); each joist is secured on a wall at one end and the other end is nailed to a longer joist in herringbone manner along the diagonal axis. With this arrangement, only one beam need span the whole space.

![Diagram](image)

**Figure 12.** The How House by R M Schindler, Los Angeles, 1925. Three pairs of concentric squares define conceptually the principal ‘blocks’ of the house. (a) The large square of 13 \( \times \) 13 modules is defined at one corner by the L-shaped concrete wall marking the street entrance and the opposite corner by the concrete wall to the terrace. The smaller concentric 4 \( \times \) 4 square defines the living room. (b) The large 9\( \frac{1}{2} \) \( \times \) 9\( \frac{1}{2} \) square includes the living room, the study, the dining room, and the terrace together with two planters. The small square defines the 1\( \frac{1}{2} \) \( \times \) 1\( \frac{1}{2} \) open shaft between the piano nobile and the floor below. (c) The large 5\( \frac{1}{2} \) \( \times \) 5\( \frac{1}{2} \) square is the terrace measured to the outer edge of the planters. The small 1\( \frac{1}{2} \) \( \times \) 1\( \frac{1}{2} \) square locates a built-in light fixture in the cantilevered porch for the terrace. The three diagrams, taken together and superimposed, show a sequence of centering points along the diagonal axis.
Two of the most striking early uses of the structural arrangement are Villard de Honnecourt’s beam arrangement and Serlio’s floor design (Yeomans, 1997). Both designs employ four-fold rotational framing within a square $C_4$. Yeomans states that the structural arrangement is deeply rooted in classical tradition and is also common to carpenters in the modern period for practical and aesthetic purposes. The structure of the How House appears to be a modern variation of the Serliian arrangement. Having a sound knowledge of structure and architectural history (Giella, 1985), Schindler may well have adopted or transformed this structural idea to suit his own purposes. Otherwise it may be his own invention. In any event he had to solve the problem of patterning the exposed structure of a square ceiling set on the diagonal, and his solution is witful both aesthetically and structurally (figure 11).

![Diagram](image)

**Figure 13.** The How House by R M Schindler, Los Angeles, 1925. Visual boundaries to the perceived spaces are defined by partial squares. (a) The square defines the living room from the external corner of the living room to two opposing adjacent walls, one for the fireplace and the other for the stairwell. (b) The square is defined by the glazed doorways to the dining room and study from the living room, and the walls to the gallery above. (c) The square defines the concrete walls in the corner of the terrace. (d) The square defines the main volume of the piano nobile which includes the external walls to the living room, dining room, and study.
Various squares along the diagonal axis guide the overall conceptual composition in the piano nobile (March, 1994b). One analytic approach is to recognize that there are situations where two squares overlap and share the same centerpoint. In contrast to the static uses of symmetry in classical architecture, translating centerpoints along the diagonal axis gives movement to the whole spatial composition (figure 12).

Another approach is to examine partial squares along the diagonal axis which define the spatial organization and the perceived boundaries (figure 13).

**Conclusion**
A method of subsymmetry analysis has been outlined and some architectural examples have been examined to demonstrate the uses of symmetry in formal composition. Each example shows unique application of symmetrical operations. The Pantheon turns out to be Hadrian’s architects’ awareness of point symmetry with an emphasis on dihedral symmetries and the orthogonal axes. In the 20th century Frank Lloyd Wright shows a keen understanding of symmetrical possibilities in his use of cyclic symmetries and diagonal axes in a decorative design of 1902. R M Schindler in 1920 demonstrates a masterful understanding of symmetrical organization in his library project where the symmetries, as in the Pantheon, are developed around a single central point. In a development of these ideas in the How House, Schindler explores the possibility of having more than one center of symmetry in the same composition. The shift from classical modes of composition to the modern can be summarized neatly in terms of three flooring designs (figure 14). The first is the marble floor pattern of the Pantheon. The next two are early 20th century: one by Peter Behrens dated 1910 for a member of the Deutsche Werkbund; and the other is the design for the linoleum flooring in the kitchen of the How House by R M Schindler.

The design of the Pantheon floor involves a play on platonic and Euclidean relationships: doubling the square, and a comparison of a square with its inscribed circle (Stierlin, 1984, page 88). The alternating pattern of square and circle induces a strong visual diagonality on the otherwise orthogonal scheme. In this way the multiple subsymmetries of the rotunda are reinforced through ornamentation. Peter Behrens’ design also toys with circles and squares but in a naive repetitive manner compared with the Roman example (Durrant, 1986, page 242). Schindler evokes the diagonality of the How House in his design (Berns, 1997, page 76). He alternates the figure and ground in such a way as to invite several readings of the design: one of which indicates

![Figure 14](image)

*Figure 14.* On the left is the floor pattern of the Pantheon in dark and light marble; in the center, the design by Peter Behrens for the Delmenhorster Linoleum fabrik; and on the right, the design from the original fragment of linoleum in the kitchen of Schindler’s How House.
the L-shapes noted in the house itself, and another which appears to be a series of black-and-white hollow squares, also a theme observed in the analysis above.

If Behrens illustrates the poverty of symmetry by reducing it to centrality in the motif and unrelieved repetition, then it is not surprising that some modernists wrote polemics against symmetry (March, 1979). On the other hand, Schindler (like Wright) infuses new playfulness into his designs precisely through tapping into unlimited resources of symmetry.

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